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NORMAL MATTER STORAGE OF ANTIPROTONS

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ABSTRACT

Various simple issues connected with the possible storage of $\bar{p}$ in relative proximity to normal matter are discussed. Although equilibrium storage looks to be impossible, condensed matter systems are sufficiently rich and controllable that nonequilibrium storage is well worth pursuing. Experiments to elucidate the $\bar{p}$ interactions with normal matter are suggested.

INTRODUCTION

Although it is technically possible, the confinement of $\bar{p}$ as a unneutralized plasma in electromagnetic traps makes no sense for energy storage because the energy density of the required magnetic field is equal to or greater than the rest mass energy density of the confined $\bar{p}$. This is called the Brillouin limit.[1] (Of course, such storage of $\bar{p}$ for other purposes, as discussed elsewhere in this workshop, makes a great deal of sense.) Therefore, energy storage must be achieved by neutralizing the $\bar{p}$ charge either directly with e+ (antihydrogen formation) or indirectly in condensed matter. Both methods confront challenging scientific questions of intrinsic interest. (See the papers by J. B. A. Mitchell and W. C. Stwalley for antihydrogen formation.)

BENEFITS

A. Space charge screening (dense storage)

The density of unneutralized $\bar{p}$ that can be stored in macroscopic electromagnetic traps is ultimately limited by space charge, which must be confined by a magnetic field. A $\bar{p}$ density of only 2.5 10^{13} cm^{-3} will create a pressure of 100 atmospheres at the surface of a spherical volume 1 cm in radius. Of course, adding positrons to create charge neutrality would be a solution, but, in principle, a deficit of electrons in condensed matter would achieve the same thing, so long as the integrity of the normal matter structure was not affected. Removing only one electron per 100 atoms would be enough to neutralize a $\bar{p}$ storage density of about 10^{22} cm^{-3} in normal matter.

B. Low energy

Although any efficient $\bar{p}$ production process now imagined produces the product at very high kinetic energies, this is an impractical state for storing 10^{22} $\bar{p}$ at any density, and especially so at high density. Even at the modest energy of 10 keV the kinetic energy of 10^{22} $\bar{p}$ is equivalent to a gram of matter moving at a velocity of 178 km/sec, about 16 times the escape velocity from earth. Should the end use of the $\bar{p}$ require kinetic energies higher than thermal it is a relatively simple matter to accelerate them compared to deaccelerating them, for which a well-defined beam requires the controlled compression of phase space. Condensed matter storage of $\bar{p}$ would be intrinsically low energy storage as would be condensed anti-hydrogen, an attractive alternative.

C. Robustness (vacuum requirements)

Without a specific mechanism in mind it is impossible to predict how robust condensed matter storage would be with all the attendant equipment. However, experience has shown that the condensed matter version of devices, from VLSI chips to IR detectors, exhibit high reliability. Paradoxically, condensed matter storage would eliminate the requirement for ultra high vacuum since the migration of impurities through a solid is easier to control than their migration through empty space. This is particularly important for low-energy $\bar{p}$ storage for which the vacuum requirements become severe because of the increased cross section for annihilation.

KNOWN LIMITS TO STABILITY

A. Lieb's theorems

Are there any fundamental reasons why no possible combination of matter and antimatter can be stable, i.e., stable intrinsically and in equilibrium? The answer, unfortunately, is yes, and is provided by Elliott Lieb's theorems on the stability of normal matter.[2] Lieb proves that atoms are stable because of the uncertainty principle, that bulk matter is stable because of the Pauli principle for fermions (which leads to a stronger uncertainty principle), and that thermodynamics is possible because of screening (which permits charge neutrality in bulk matter). These theorems apply with equal force, of course, to antimatter, so that is stable, too. However, combinations of matter and antimatter necessarily involve an interface where there is no Pauli principle between the electrons and positrons and, hence, no stability.

There are two ways around Lieb's theorems. One way is to note that the role of the Pauli principle was crucial only for the leptonic component of matter, not the hadronic. (Solids containing only nuclei having integral spin (bosons) are quite stable.) The fact that protons and neutrons are fermions may be relevant to the stability of neutron stars, but not to ordinary matter. Therefore, the theorems do not strictly address the problem of $\bar{p}$ stability in normal matter. The other escape is to accept the impossibility of equilibrium stability but work for either nonequilibrium stability in steady state (the basis of $\bar{p}$ storage rings) or a long decay time (the basis of some electromagnetic traps).

B. Intrinsic attraction through induced polarization

If $\bar{p}$ without e^+ avoid Lieb's objections, what is the problem? The problem is not only that Earnshaw's theorem[3] prohibits trapping in a static electric field but that a $\bar{p}$ induces attractive electric dipole forces in all neutral, equilibrium matter. Therefore, a thermalized $\bar{p}$ will be attracted to the nearest positive ion in a solid, or to a neutral atomic site, where it will become captured in an atomic orbit and then cascade down the atomic energy levels until annihilation occurs with the nucleus.

C. The question of feasibility

In view of these daunting obstacles what hope can there be for $\bar{p}$ storage in condensed matter? Without its technological importance, shared with $\bar{H}$ condensation, as the ultimate means for energy storage, the problem would be dismissed as too difficult. However, until it can be <u>proven</u> impossible, with the rigor of Lieb's theorems or the second law of thermodynamics, it must be assumed possible because of the astonishing variety of complex and subtile effects that condensed matter continues to reveal. Once a $\bar{p}$ reaches thermal energies its behavior can be dominated by these electromagnetic effects as long as it remains outside the vicinity of nuclei. From another viewpoint, every mole of condensed matter contains 10^{24} force-free positions for $\bar{p}$ -- unstable, though, in one or more directions. To dynamically stabilize a small fraction of these is "all" that is needed.

DOWN-SCALING MACROSCOPIC TRAPS

A. Penning traps to microfabrication to Stark saddles

Can proven macroscopic traps be scaled down to microscopic size? The reduction in size may be carried quite far, although true atomic analogs are not yet known.

Consider a standard Penning trap with electrodes along equipotential lines of the electric potential[4]

$$\Phi = A(x^2 + y^2 - 2z^2)$$

in an uniform magnetic field H the z direction. If z_0 and $x^2 + y^2 = r_0^2$ are the locations of the electrodes then A is related to the applied voltage V by

$$A = \frac{V}{2z_0^2 + r_0^2}$$

The three characteristic frequencies are

$$\omega_m = 2cA/H \quad , \quad \textit{magnetron} \ (c = \textit{velocity of light})$$

$$\omega_z = 2 \quad \frac{eA}{m} \qquad \textit{harmonic}$$

$$\omega_c = \frac{eH}{mc} \qquad \textit{cyclotron}$$

There will be a maximum number of charges the trap can hold. For simplicity, take this number N to be that which would cancel a fixed fraction f of the applied field at z_0 when all charges are at the center of the trap.

$$\frac{Ne}{z_0^2} = f4Az_0$$

The effective charge density ρ is then

$$\rho = \frac{N}{2\pi z_0 r_0^2} = \frac{f2z_0^2V}{\pi e r_0^2(2z_0^2 + r_0^2)}$$

Consider how this density scales with size The voltage V must not produce an electric field strength above the value for dielectric or vacuum breakdown, so the scaling of V will be taken as

$$V = E_0 z_0$$

where E_0 is a (safe) constant Taking the ratio z_0/r_0 to be constant it is seen that ρ scales as $1/z_0$, which implies smaller is better Assuming constant H, the frequencies scale as

$$\omega_m \propto 1/z_0$$

$$\omega_z \propto 1/\sqrt{z_0}$$

$$\omega_c = const$$

One limit to smaller traps is the lower critical field for stability.

$$H_c = c\,\frac{2mV}{z_0^2}$$

This is equivalent to requiring $\omega_m \leq \omega_z/\sqrt{8}$ if $r_0 = \sqrt{2}\, z_0$. Clearly, H_c scales as $1/\sqrt{z_0}$, so the maximum attainable magnetic field will set a lower limit on the trap size, and an upper limit on the density. If $E_0 = 10^4$ volts/cm and $H_c = 10^6$ gauss (thanks to the new superconductors), then

$$(z_0)_{min} = 2 \quad 10^{-4}\, cm$$

and

$$(\bar{p})_{max} = f\,2\ 6\ 10^{13}\, cm^{-3}$$

This is in the relm of microfabrication, but not quite quantum mechanics. Note that the space charge in each small trap can be neutralized, so there is no build-up of charge as the number of traps is increased. If the total volume of a small trap is five times its storage volume and $f = 0.2$, then a cubic meter of these would contain $10^{18}\bar{p}$. (The Brillouin limit corresponds to $f = 1$.) The required voltage would be a very modest 2v

Although miniaturized, these traps are still classical and therefore require a high vacuum and a high, externally imposed magnetic field whose energy density is comparable to the rest energy of the trapped $\bar{p}$.

Quantum mechanics does enter for further miniturization, where an atomic analog to the Penning trap exists, as pointed out by Charles Clark, et al [5] The so-called Stark saddle is the force-free location of a charge subject to both an external electric field and the electric field of an ion. This position is unstable only in the direction toward (or away) from ion. Applying a magnetic field perpendicular to the unstable direction leads to closed, classical orbits for the charge's motion around the saddle point. Unlike the Penning trap, in which a magnetic field H perpendicular to a plane with E field instability results in stability, the Stark saddle trap has H perpendicular to a plane with an E field saddle, and the result is metastability. That is, the classical orbits are unstable to small perturbations and correspond to resonances rather than true bound states. For an external field of 5 kV/cm the Stark saddle lies over 600 Å from the ion, so this a phenomenon for gases, not solids.

B. Storage ring to channel ring

The other common macroscopic storage medium for $\bar{p}$ is the storage ring. In principle, it, too, can be down-sized, and this important subject is covered by D. Cline.[6] Going to even smaller scales, is there an atomic analog? An obvious one would be channeling in a crystal, which would demand some unusual fabrication to make a closed path. It is also possible to imagine (but undoubtedly more difficult to realize) channeling in a straight path with perfect reflection at each end. If lossless channeling occurs for only certain ranges of $\bar{p}$ energy the reflection region would have to be tailored with a varying impedance to minimize turning-point losses. Although no $\bar{p}$ channeling experiments have been done, π^- channeling has been observed in the curious configuration of a helical spiral around lines of atoms.[7]

CONDENSED MATTER TRAPS

A Generic leakage

Whatever the mechanism for achieving $\bar{p}$ traps there will be a relationship between the size and depth of the trap and the leakage rate to neighboring nuclei where annihilation will occur. This relationship is illustrated here with a simple model

Let the trap be an equivalent three-dimensional square well of depth $-V_0$ and radius r_0. Bound states of energy $-\varepsilon V_0$ will exist if

$$g = r_0\sqrt{2mV_0}/\hbar > \pi/2$$

where m is the $\bar{p}$ mass. The wavefunction of the lowest bound state (s wave) will extend outside the well leading to a probability density

$$|\psi(r)|^2 = \frac{A^2}{r_0^3}\left(\frac{r_0}{r}\right)^2 \sin^2\theta\, e^{-2\sqrt{\varepsilon}\, g(r/r_0-1)}$$

for $r \geq r_0$, where A is a normalization constant and $\theta = \sqrt{1-\varepsilon}\, g$.

This probability density will extend to the nearest nucleus and give an annihilation decay rate. To estimate the distance r_d at which the decay rate is 1 per year assume for simplicity that the proton at r does not change $|\psi|^2$, i.e., the Born approximation, and compare the density $|\psi|^2$ with the 1s state density at the origin of protonium $|\Psi_{10}(0)|^2$, which has a decay rate[5] of $5 \cdot 10^{18}\,s^{-1}$ That is, solve the following equation for r_d:

$$|\psi(r_d)|^2/|\psi_{10}(0)|^2\, 5 \cdot 10^{18} = 10^{-8}$$

The results for r_d corresponding to various well strengths are shown in the table below where a trap of radius $r_0 = 10^{-8}$ cm is assumed

g	V_0(ev)	ε_{min}	$r_d \cdot 10^{-8}$ cm	$r_u \cdot 10^{-8}$ cm
π	.81	0.458	8.9	9.5
3/2π	1.82	0.704	5.3	5.6
3π	7.3	0.909	2.9	3.0
10π	80.8	0.991	1 5	1.6

The table shows that a physically reasonable trap could hold $\bar{p}$ for a year within a few Ångstroms of normal matter, although there is the obvious trade-off between shallow traps (easier to achieve) and larger spacings (harder to achieve)

Actual annihilations will most likely proceed through the capture of a $\bar{p}$ in an atomic state rather than by direct annihilation. To estimate the critical radius r_a for an atomic capture of 1 per year, set the particle flux leaving the well equal to $|\psi(r)|^2$ times $\hbar k/m$, where k is the wavenumber of the $\bar{p}$ in the atomic potential, $k = \sqrt{2m\Delta E}/\hbar$, $\Delta E = me^4/4\hbar^2$ (energy of lowest $\bar{p}p$ state). This flux multiplied by the protonium area $2\pi a_0^2 = 2\pi(2\hbar^2/me^2)^2$ gives the approximate rate of capture (a quantum mechanical engineering estimate!). That is, solve the following equation for r_a,

$$|\psi(r_a)|^2 \hbar k/m \; 2\pi a_0^2 = 10^{-8}$$

which has the same form as the previous equation used to find the critical radius r_d for direct $\bar{p}p$ annihilation. Numerically, atomic capture is 10 times more likely, which results in slightly larger r_a as shown in the previous table.

Electrons will not be trapped in these wells unless $\sqrt{m_e/m_p}\, g > \pi/2$, i.e., $g > 21.4\,\pi$, much larger than considered above. Even then the traps could not be "poisoned" by electron filling because the heavier particle always wins the competition for the same trap, otherwise the stability of $\bar{p}$ in normal matter would be easier to achieve. Likewise, muons can not be trapped until $g > 1.5\pi$ which is reasonably close to optimum trap depths for $\bar{p}$. Therefore, μ^- would be a relatively cheap substitute for $\bar{p}$ when testing trap concepts.

3. Polarons

A polaron is a charged particle in a crystal with the accompanying lattice distortions it induces.[9] In the extensive literature on polarons the particle is usually an electron, but, in principle, it could be an interstitial proton or $\bar{p}$. The heavier the particle the more dense the phonon cloud around it, and it is known that protons can even be trapped. By itself, the observed phenomenon of proton trapping does not prove that $\bar{p}$ trapping will likewise occur, but it is a promising entreé for theorists into the behavior of $\bar{p}$ in alkali halide crystals. The polaron phenomenon is responsible for the effective attraction between electrons that leads to superconductivity, which demonstrates its capability for non-trivial effects.

C. Excitons

Excitons are electron-like bound states[9] and are of interest to the $\bar{p}$ system because they are the quantum unit of the polarization field, and it is precisely on questions of induced polarization that the $\bar{p}$ storage problem hangs. If excitons were large objects spread over many lattice spacings their relevance would be minimal, but, in fact, they may occur localized on one atom. Again, as with polarons, the exciton is a concept with a rich literature of special relevance to theoretical studies of $\bar{p}$ trapping concerned with nonlinear polarization or screening of the $\bar{p}$ charge by holes.

SPECIAL EFFECTS

The following mechanisms are examples of new discoveries and ideas that could lead to a breakthrough in the $\bar{p}$ storage problem.

A. Suppressed barrier penetration

In a study of quantum mechanical tunneling Michael Nieto, et al.,[10] have found that a wave function in a higher-energy well will not necessarily tunnel to a lower-energy well, even in an arbitrarily long time, if there is not dissipation or coupling to other modes. The probability of quantum tunneling is a critical function of the shape of the barrier potential, not just its average height. Temporal modulation of the barrier would seem to be another promising means to control tunneling rates.

B. Quenched decay

E. J. Robinson[11] has shown that the well-known exponential decay rate of unstable states can be changed substantially, and even approach zero, if the product of the decay belongs to a continuum that is itself unstable.

C. Self-trapping

Using a model relevant to the quantum diffusion of muons and protons inside metals and at surfaces, F. Guinea, et al.,[12] discovered a transition from diffusive dynamics to a self-trapped state at a critical value of coupling to the environment This trapping is not related to the self-trapping of polarons mentioned above

D Dynamic localization

D. Dunlap, et al.,[13] calculated the quantum mechanical motion of a charged particle in a lattice under the action of time-dependent electric fields and found a new phenomenon whereby the moving particle became localized within one lattice spacing. Since this localization occurs exactly at a lattice site, it is not directly applicable to p̄, which need to avoid lattice sites. Nevertheless, the concept of dynamic localization, or creating effective traps by parametric modulation, seems a promising mechanism to apply to energy saddle points of p̄ in normal matter.

E. Two-level systems

The properties of the two-level quantum system have been recently studied by many people (see the review by A. J. Leggett, et al [14]) and many exact results are known about the tunneling of a particle between two wells in a dissipative environment. The conditions for the particle being localized in one well, decaying, or oscillating between the wells have been dilineated in detail.

EXPERIMENTS WITH p̄ IN CONDENSED MATTER

The easy availability of low-energy p̄ will be a strong impetus to perform experiments in condensed matter. Although most of these will not be intended to test proposed storage mechanisms, the experience gained will build, nevertheless, an invaluable technical base for the critical evaluation of storage feasibility Predictions of the benefits of condensed matter experiments have consistently missed the most wonderful discoveries, a recent example being the high temperature superconductors, so the best strategy is to encourage experimentation. The role for theory should be to suggest and interpret, not to proscribe.

A. Atomic decay and strong interactions

The x-rays emitted as a $\bar{p}$ cascades down the atomic states when it is captured by a nucleus are recognized as a powerful diagnostic for studying the $\bar{p}p$ and $\bar{p}N$ interactions at low energies.[15] The effect of the strong interaction shows up as a reduction in the intensity of the last observable transition and as a shift (of order 1. keV) in the low lying atomic states.[16-19] There is also a hyperfine splitting between singlet and triplet s-states (of order 1/4 keV) due to the spin-dependent $\bar{p}N$ interaction.[20,21] X-rays from np → 1s and nd → 2p transitions of protonium have been seen[16] and they confirm predictions of strong interaction effects. Other groups have seen strong interaction effects in $\bar{p}N$.[18,19] Also, the preferential capture and different atomic cascades of the various negative particles, π^-, K^-, Σ^-, and $\bar{p}$ reveal much about low energy cross sections and metastable states.[22,23] If the atomic deexcitation energy is resonant with an appropriate nuclear E2 excitation it is also possible for radiationless transitions to occur (the atom deexcites by exciting the nucleus).[24,25] This can provide otherwise inaccessible information about the nuclear potential.

B. Channeling

Unlike the case with μ^+ there is little known about μ^- channeling,[26,27] which would give useful information about the prospects with $\bar{p}$. The reason for this is the lack of a μ^- beam or a μ^- source within the solid. The obvious source, a π^-, is captured by a nucleus (as a $\bar{p}$ would be, normally). However, channeling of π^- themselves has been observed and the paths appear to be helical spirals, as mentioned above.[7] The fact that π^- can channel is most interesting because it is doing so in a lattice of absorbers rather than repellers, and therefore falls outside the usual analysis of channeling.[28]

C. Potpourri

Because condensed matter is so complicated, many of the scientific breakthroughs arise from unexpected experimental results rather than theoretical discoveries. Therefore, as low-temperature $\bar{p}$ became cheaper they undoubtedly will be inserted in various materials if only "to see what happens." Some of the materials that have been suggested are discussed below.

1 Superfluid ^{4}He

Superfluid ^{4}He is one of the most studied and best understood of condensed matter systems. The atoms are the most quiescent of all, with about 10% of them being in a Bose condensate having strictly zero momentum. The atoms also have the highest ionization potential, 24.6 eV, and are correlated into a macroscopic quantum state from the size of the container to an interatomic spacing.

What would $\bar{p}$ do in such an exotic material? It has been suggested they might form "bubbles" or self-containing cavities in the liquid, as other impurities do, such as electrons[29] and even positronium.[30] Bubble formation, however, does not seem likely because (1) there is no help from the Pauli principle (as with electrons) in pushing away the electron densities of nearby helium atoms and (2) the larger $\bar{p}$ mass reduces the $\bar{p}$ localization energy (zero-point motion) to a scale comparable with the interatomic spacing. Perhaps there will be a barrier to $\bar{p}$ atomic capture because the intermediate state involves a free electron which must form a bubble, which costs 1.3 eV? Alas, there seems no reason to exclude He$-$ as an intermediate state, for which there is no significant barrier. Impurities are also attracted to the cores of quantized vortex lines, but there is no obvious advantage for $\bar{p}$ stability at such a location. Nevertheless, it will be interesting to see what does happen to $\bar{p}$ in superfluid ^{4}He.

Another interesting facet of ^{4}He behavior is its surface, which is microscopically smooth, adjoins a vacuum of arbitrary "hardness" at sufficiently low temperature, and can support sheets of surface charge on either side.[31,32] In particular, electrons on the vapor side can be held against the helium surface by applying an electric field; they do not penetrate the surface because of the relatively high energy required to make the bubble state mentioned earlier. Because this electronic surface charge density can be substantial and can be excited in various plasma modes, the possibility exists of finding electron-$\bar{p}$ states that are bound to the surface but have negligible $\bar{p}$ density at the surface. In effect, the $\bar{p}$ would be trapped between the external electric field and the electronic surface charge, which in turn is repelled from the surface by the Pauli principle. One could also introduce other charged species of heavier mass, such as H^- or D^-. As mentioned in the general remarks above, such trapping would have to occur in an excited state.

Even quite small or dilute trapped surface states would be interesting as possible nucleation sites for cluster ion formation, for which the problem is to find a coupling (to normal matter) that can carry away the condensation energy. (See the paper in these proceedings by W. C. Stwalley.)

2. Degenerate liquid ^3He

This is mentioned more to illustrate a general approach to trapping -- prohibit the formation of intermediate states necessary for decay -- than to suggest it will work in this specific instance. At low temperature, the ^3He atoms are in the ground state of a Fermi liquid, which means that all momentum states less than the Fermi momentum, $k_F \approx 0.3$ Å^{-1} are filled. To the extent that intermediate states of the $\bar{p}$ decay process require the scattering of ^3He atoms into states with $k < k_F$ the decay will be suppressed. The problem is that $\bar{p}$ is a localized perturbation, and it would seem to have no difficulty in confining its interactions to wavelengths $\lambda < 2\pi/k_F$.

3. Superconductors

Many of the features of quantum coherence apply to both superconductors and superfluids (i.e., a superconductor is a charged superfluid in solid neutralizing background Electric and magnetic fields are shielded quite effectively in superconductors over distances comparable to the penetration depth, a length scale present only in charged superfluids and typically having a magnitude of many lattice spacings. Thus, one cannot expect known superconductors to shield and stabilize a $\bar{p}$ in any obvious way on an atomic scale, but what <u>will</u> happen is not clear either since the origin of the effective electron attraction, which gives rise to superconductivity, is a subtle and delicate interplay of electronic and lattice properties both of which are disturbed by a $\bar{p}$ A best guess now is that the influence of a $\bar{p}$ impurity will be too local to probe superconductivity, although it could give information on other electronic structure

4 Semiconductors

A $\bar{p}$ is attracted to positive charge If there were an effective source of positive charge, other than protons, one could expect $\bar{p}$ trapping In many respects, especially involving dynamics and transport, the absence of electrons is equivalent to positive charge. This "positive" charge can be either delocalized as holes in a conduction band or localized at ionic lattice vacancies and certain crystal imperfections. Of course, such pseudo positive charge cannot violate the laws of electrostatics, and the earlier remarks on the absence of ground state stability still hold Nevertheless, the existence of localized exciton states of the $\bar{p}$-hole system seem possible, in principle, and the model could serve as a fruitful paradigm

REFERENCES

1 C F Driscoll, in <u>Low Energy Antimatter</u>, edited by D B Cline (World Scientific, Singapore, 1986), p 184

2 E H Lieb, Rev Mod Phys <u>48</u>, 553 (1976)

3 S Earnshaw, Trans Cambridge Philos Soc <u>7</u>, 97 (1842)

4 H G Dehmelt, Adv At Mol Phys <u>3</u>, 536 (1967)

5. C W Clark, E. Korevaar, and M G Littman, Phys Rev Lett. 54, 320 (1985)

6 D. Cline, these Proceedings

7 T. H Braid, D. S Gemmell, et al. Phys Rev B19, 130 (1979)

8. B R Desai, Phy . Rev 119, 1385 (1960). This early prediction agrees closely with the recent measurement of the width of the 1s level of protonium C A Baker et al in Physics with Antiprotons at LEAR in the ACOL Era, edited by U. Gastaldi et al (Frontiéres, Gif-sur-Yvette, France, 1985), p. 219

9 C. Kittel, Quantum Theory of Solids Wiley, NY, 1966).

10. M M. Nieto, et al., Phys Lett. 163B, 336 (1985)

11. E J Robinson, Phys Rev. Lett. 57, 1281 (1986).

12 F Guinea, V Hakim, and A Muramatsu, Phys. Rev Lett. 54, 263 (1985).

13. D H Dunlap and V M. Kenkre, Phys. Rev B 34, 3625 (1986)

14 A J Leggett, et al., Rev Mod. Phys 59, 1 (1987)

15 T von Egidy, Nature 328, 773 (1987)

16 S Ahmad, et al, Phys. Lett. 157B, 323 (1985)

17 V S Popov, A. E Kudryavtsev, and V. D Mur, Sov Phys. JETP 50, 865 (1979) [Zh. Eksp. Teor Fiz 77, 1727 (1979)]

18 G Backenstoss, et al, Phys Rev Lett 41B, 552 (1972)

19 P D Barnes, et al, Phys Rev Lett 29, 1132 (1972)

20 J M Richard and M E Sainio, Phys Lett. 110B, 349 (1982)

21 A M Green and S Wycech, Nucl Phyc A377, 441 (1982)

22 J G Fetkavich, et al, Phys Lett 35B 178 (1971)

23 G A Baker, Phys Rev 117, 1130 (1960)

24 M Leon et al, Nucl Phys A322, 397 (1979)

25 C J Batty, Nature 322, 487 (1986)

26 D S Gemmell, Rev Mod Phys 46, 129 (1974)

27. D. V Morgan. editor. Channeling; Theory; Observations and Applications (Wiley. NY. 1973)

28. A W. Sáenz. J. Math Phys. 26. 1925 (1985)

29. C. G. Kuper. Phys Rev. 122. 1007 (1961)

30. R. A. Ferrell. Phys. Rev. 108. 167 (1957).

31. C. C. Grimes. Surf Sci 73. 379 (1978)

32. G. A. Williams and J Theobald. Phys Lett A77. 255 (1980).